SEMESTER-1 (Period-III)

TOPIC

3

Momentum and its Conservation

P11CH3

3.1. MOMENTUM (LINEAR)

Newton introduced the concept of momentum to measure the quantitative effect of force.

The total quantity of motion possessed by a moving body is known as the ***momentum of the body***. It is the product of the mass and velocity of a body. It is denoted by $\vec{p}$. $\vec{p} = m\vec{v}$

Since mass m is always positive therefore the direction of $\vec{p}$ is the same as that of $\vec{v}$.

In magnitude, $|\vec{p}| = m|\vec{v}|$ or $p = mv$

Since velocity is a vector and mass is a scalar therefore momentum is a vector. Again, $\vec{p}$ has same direction as that of $\vec{v}$ because m is always positive.

The cgs and SI units of momentum are g cm s^{-1} and kg m s^{-1} respectively.

The dimensional formula of momentum is [MLT $^{-1}$].

(i) When m is constant, $p \propto v$. This is shown in Fig. 3.1.

(ii) When v is constant, $p \propto m$. This is shown in Fig. 3.2.

(iii) When p is constant, then $v \propto \frac{1}{m}$. This is shown in Fig. 3.3.

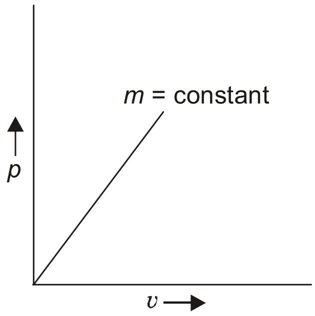


Fig. 3.1

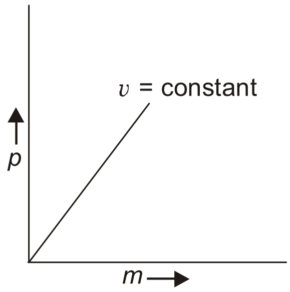


Fig. 3.2

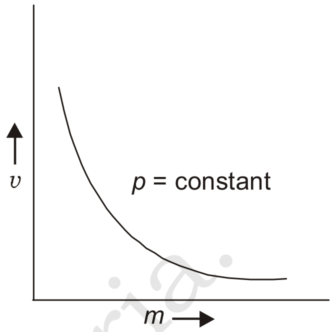


Fig. 3.3

Conceptual Problem 1. *A car and a scooter are travelling with the same speed. Which of the two has greater momentum?*

Ans. Let M and m be the masses of the car and scooter respectively. Let p_c and p_s be their respective momenta. Let v be the speed of both scooter and car.

Now, $$p_c = Mv \quad \text{and} \quad p_s = mv, \quad \frac{p_c}{p_s} = \frac{Mv}{mv} = \frac{M}{m}$$

$\because \quad M > m \qquad \therefore \quad p_c > p_s$

So, the momentum of the car is greater than the momentum of the scooter.

Conceptual Problem 2. *A car and a scooter have the same momentum. Which of the two has greater speed?*

Ans. In this case, $p = Mv_c = mv_s$

where v_c and v_s are the speeds of the car and scooter respectively.

Now, $$\frac{v_c}{v_s} = \frac{M}{m}. \quad \because \quad m < M \qquad \therefore \quad v_c < v_s$$

So, the speed of the car is less than the speed of the scooter.

Conceptual Problem 3. *Establish a general relation between momentum p and kinetic energy* E_k.

Ans. $$p = mv \; ; \; p^2 = m^2v^2$$

or $$p^2 = 2\,m\,\frac{1}{2}mv^2 = 2m\,E_k \quad \text{or} \quad p = \sqrt{2m\,E_k}$$

3.2. NEWTON'S SECOND LAW OF MOTION

(*i*) Statement. ***The time rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the force.***

(*ii*) Explanation of Newton's second law. The statement can be divided into the following two parts:

(*a*) *The time rate of change of momentum of a body is proportional to the impressed force.*

A force acting on a body produces a certain change in the momentum of the body. When the given force is doubled, the 'change in momentum' of the body is also doubled. So, as the applied force is increased, the rate of change of momentum of the body is also increased.

(*b*) *The change of momentum takes place in the direction of the force.*

Consider a body to be at rest. When a force is applied on this body, the body will begin to move in the direction of the force. If a force is applied on a moving body in the direction of motion of the body, then there is an increase in the momentum of the body. However, if the force is applied on a moving body in a direction opposite to the direction of motion of the body, then there is a decrease in the momentum of the body.

(*iii*) Formula for force. Let a constant external force $\vec{F}$ acting on a body change its momentum from $\vec{p}$ to $\vec{p} + d\vec{p}$ in time interval dt. Then, the time rate of change of linear momentum is $\frac{d\vec{p}}{dt}$.

According to Newton's second law of motion,

$$\frac{d\vec{p}}{dt} \propto \vec{F} \quad \text{or} \quad \vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F} = k\frac{d\vec{p}}{dt}$$

Here k is a constant of proportionality. The value of k depends upon the units selected for the measurement of force. In both SI and cgs system, the unit of force is so chosen that $k = 1$.

$$\therefore \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

3.3. IMPULSE

The effectiveness of a force in producing motion depends not only upon the magnitude of the force but also on the time for which the force acts. When a large force acts for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured. **The total effect of force is called impulse.** It may also be defined as a measure of the action of force. It is a vector quantity and is denoted by $\vec{J}$. It is the product of force and the time for which the force acts.

Suppose a force $\vec{F}$ acts for a short time *dt*. The impulse of this force is given by, $d\vec{J} = \vec{F}\,dt$

If we consider a finite interval of time from t_1 to t_2, then the impulse is given by,

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt$$

The right hand side of the above equation represents the impulse of varying force.

$$\vec{J} = \vec{F}\int_{t_1}^{t_2} dt = \vec{F}\left[t\right]_{t_1}^{t_2} = \vec{F}(t_2 - t_1)$$

or $\vec{J} = \vec{F}\Delta t$ where $\Delta t = t_2 - t_1$

So, the impulse of a constant force $\vec{F}$ is equal to the product of the force and time interval Δt for which the force acts.

The direction of $\vec{J}$ is the same as the direction of $\vec{F}$.

3.4. UNITS AND DIMENSIONS OF IMPULSE

In **cgs system,** the unit of impulse is dyne second or g cm s^{-1}.

In **SI**, it is measured in newton second or kg m s^{-1}.

The dimensional formula of impulse is $[MLT^{-1}]$.

3.5. IMPULSE-MOMENTUM THEOREM

Impulse is measured by the total change in momentum that the force produces in a given time.

According to Newton's second law of motion, $\vec{F} = \dfrac{d\vec{p}}{dt}$

where $\vec{p}$ is the momentum of body at any time t and $\vec{F}$ is the applied force at that time.

$$d\vec{p} = \vec{F}\,dt$$

Integrating, $$\int_{p_1}^{p_2} d\vec{p} = \int_0^t \vec{F}\,dt$$

where $\vec{p}_1$ is the momentum at $t = 0$ and $\vec{p}_2$ is the momentum at time t.

$$\left[\vec{p}\right]_{p_1}^{p_2} = \int_0^t \vec{F}\,dt \quad \text{or} \quad \vec{p}_2 - \vec{p}_1 = \int_0^t \vec{F}\,dt$$

or $$\int_0^t \vec{F}dt = \vec{p}_2 - \vec{p}_1 \qquad \ldots(1)$$

So, *the impulse of a varying force is equal to the change in momentum produced by the force.*

If the applied force $\vec{F}$ is constant, then from equation (1),

$$\vec{F}\int_0^t dt = \vec{p}_2 - \vec{p}_1 \quad \text{or} \quad \vec{F}\left[t\right]_0^t = \vec{p}_2 - \vec{p}_1$$

or $$\vec{F}\,(t - 0) = \vec{p}_2 - \vec{p}_1 \quad \text{or} \quad \vec{F}\,t = \vec{p}_2 - \vec{p}_1$$

Thus, *the impulse of a constant force is equal to the change of momentum.*

In the case of positive impulse acting on a body, there is an algebraic increase in the momentum of the body. If the impulse is zero, then there is no change in the momentum. In the case of negative impulse, there is a decrease in the momentum.

3.6. PRACTICAL APPLICATIONS OF IMPULSE

These are based on the fact that if the total change in momentum takes place in a very short time, then the force is large. If the change in momentum takes place over a longer interval of time, then the force is small.

If two forces $\vec{F}_1$ and $\vec{F}_2$ act on a body to produce the same impulse, then their respective times of application t_1 and t_2 should be such that

$$\vec{F}_1 \, t_1 = \vec{F}_2 \, t_2$$

Following are the *practical applications* of impulse.

1. *While catching a fast moving cricket ball, a player lowers his hands.* In this way, the time of catch increases and the force decreases. So, the player has to apply a less average force. Consequently, the ball will also apply only a small force (reaction) on the hands. In this way, the player will not hurt his hands.

2. *Automobiles are provided with spring systems.* When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, thereby reducing the force. This minimises the damage to the automobile. [For the same reason, buffers are provided between the bogies of a train.]

3. *China plates are wrapped in paper or straw pieces while packing.* If, during transportation, the package gets a jerk, the time of blow will be increased. This will reduce the force of blow. In this way, the china plates will be saved from damage.

4. *It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity.* This is due to the fact that the cricket ball is heavier than a tennis ball. The change in momentum is more in the case of a cricket ball than in the case of a tennis ball. As a result, more force is required to be applied in the case of a cricket ball.

5. *When a moving vehicle strikes against a wall, a large amount of force acts on the vehicle.* This is because the change in momentum is very large and is brought about in a very short interval of time. So, a large amount of force acts on the vehicle and the vehicle is damaged.

Example 1. *Force-time graph for a body is shown in Fig. 3.4. What is the velocity of the body at the end of 11 second ? Mass of the body is 7 kg. Assume the body to be starting from rest.*

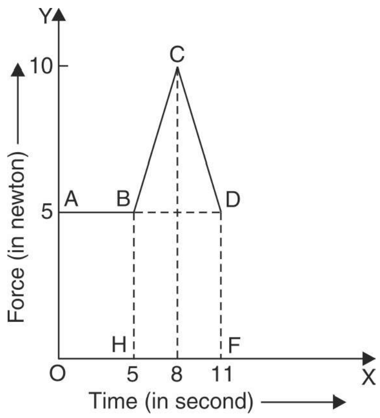


Fig. 3.4

Solution. Area ABHO = 5 × 5 = 25 units

Area BDFH = 5(11 – 5) = 30 units

Area BCD = $\frac{1}{2} \times 6 \times 5$ = 15 units

Total area under the curve = (25 + 30 + 15) units = 70 units

Since the area under F-*t* curve gives impulse *i.e.,* change in momentum,

$$\therefore \quad mv - 0 = 70 \quad \text{or} \quad v = \frac{70}{m} = \frac{70}{7} \text{ m s}^{-1} = \mathbf{10\ m\ s^{-1}}$$

Example 2. *A ball moving with a momentum of 5 kg m s^{-1} strikes against a wall at an angle of 45° and is reflected at the same angle. Calculate the change in momentum (in magnitude).*

Solution. Let $\vec{p}_1$ and $\vec{p}_2$ be the initial and final momenta respectively of the ball.

Change in momentum = $\vec{p}_2 - \vec{p}_1$

$= \vec{p}_2 + (-\vec{p}_1) = \vec{AB}$

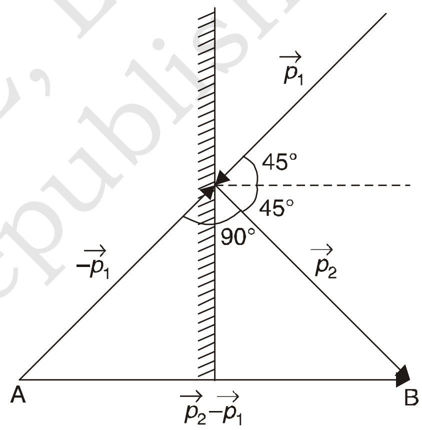


Fig. 3.5

From the Fig. 3.5,

$$AB = \sqrt{p_1^2 + p_2^2}$$

$$= \sqrt{5^2 + 5^2} \text{ kg m s}^{-1}$$

or $\quad AB = \sqrt{50}$ kg m s^{-1} = **7.07 kg m s^{-1}**

3.7. NEWTON'S THIRD LAW OF MOTION

Forces acting on a body originate in other bodies that make up its environment. This property of forces was first stated by Newton in his third law of motion:

"To every action, there is always an equal (in magnitude) and opposite (in direction) reaction."

This law may also be stated as under:

"Action and reaction are equal in magnitude, opposite in direction and act on different bodies."

Consider interaction (action and reaction) between two bodies A and B. Let $\vec{F}_{BA}$ be the force exerted by A on B and $\vec{F}_{AB}$ the force exerted by B on A (Fig. 3.6). Then, according to Newton's third law of motion,

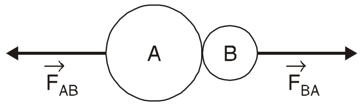


Fig. 3.6. Newton's third law of motion

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

It is clear from this equation that the two forces are equal in magnitude but opposite in direction. These forces of action and reaction act along the line joining the centres of two bodies.

One of the two forces involved in the interaction between two bodies may be called 'action' force. The other force will be called the 'reaction' force. The forces of action and reaction constitute a mutual simultaneous interaction. It cannot be said that action is the cause of reaction or reaction is the effect of action.

Newton's third law of motion leads us to a very interesting fact about forces. It is that the forces always exist in pairs. They never exist singly.

3.8. ELASTIC COLLISIONS AND ELEMENTARY IDEA OF INELASTIC COLLISIONS

A ***collision*** *is said to take place when either two bodies physically collide against each other or when the path of one body is changed by the influence of the other body.*

As a result of collision, the momentum and kinetic energy of the interacting bodies change. The forces involved in a collision are action-reaction forces, *i.e.,* the internal forces of the system. So, the total momentum is conserved. Also, the total energy is conserved.

Elastic Collision. *A collision is said to be an elastic collision if both the kinetic energy and momentum are conserved in the collision.*

During collision, the bodies are deformed. However, they regain their original shape completely if the collision is elastic. The mechanical energy is not converted into any other form of energy. In an elastic collision, the forces of interaction are conservative in nature.

Inelastic Collision. *A collision is said to be an inelastic collision if the kinetic energy is not conserved in the collision. However the momentum is conserved.*

The kinetic energy lost in the collision appears in the form of heat energy, sound energy, light energy, etc. The forces of interaction in an inelastic collision are non-conservative in nature.

If a ball is dropped from a certain height and the ball is unable to rise completely to its original height, then it would mean that ball has lost some kinetic energy (which would appear as heat energy). This would mean that collision is an inelastic collision.

Characteristics of Inelastic Collisions

(i) Kinetic energy is not conserved. ***(ii)*** Total energy is conserved. ***(iii)*** Momentum is conserved. ***(iv)*** Some or all of the forces involved in the collision are non-conservative. ***(v)*** A part of the mechanical energy is converted into heat, light, sound, etc.

3.9. HEAD-ON ELASTIC COLLISION [ONE-DIMENSIONAL ELASTIC COLLISION]

One-dimensional elastic collision *is that elastic collision in which the colliding bodies move along the same straight line path before and after the collision.*

Consider two bodies A and B of masses m_1 and m_2 respectively moving along the same straight line in the same direction [Fig. 3.7]. Let $\vec{v}_{1i}$ and $\vec{v}_{2i}$ be their respective velocities such that $|\vec{v}_{1i}| > |\vec{v}_{2i}|$.

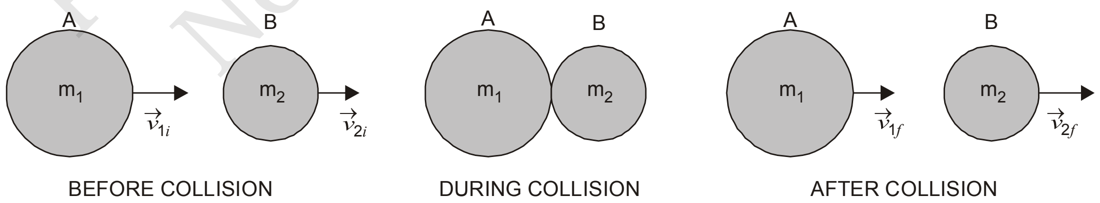


Fig. 3.7. One-dimensional elastic collision

The two bodies will collide after some time.

During collision, the bodies will be deformed in the region of contact. So, a part of the kinetic energy will be converted into potential energy.

The bodies will regain their original shape due to elasticity. The potential energy will be reconverted into kinetic energy. The bodies will separate and continue to move along the same straight line in the same direction but with different velocities.

- In an elastic collision, the kinetic energy onservation does not hold at every instant of ollision. It holds after the collision is over.
- Total linear momentum is conserved both in lastic and inelastic collisions.
- Total linear momentum is conserved at each instant of elastic and inelastic collisions.
- Total energy is conserved in all collisions.

Let *$\vec{v}_{1f}$ and $\vec{v}_{2f}$ be the velocities of A and B respectively after the collision.

Applying the law of conservation of momentum,

total momentum before collision = total momentum after collision

$$\therefore \quad m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \text{ (in magnitude)}$$

or $$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i}) \quad \ldots(1)$$

Since the collision is elastic therefore kinetic energy will be conserved.

$\therefore$ Kinetic energy before collision = Kinetic energy after collision

$$\therefore \quad \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

or $$m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

or $$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$$

or $$m_1 (v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2 (v_{2f} + v_{2i})(v_{2f} - v_{2i}) \quad \ldots(2)$$

Dividing (2) by (1), we get $v_{1i} + v_{1f} = v_{2f} + v_{2i}$

or $$v_{1i} - v_{2i} = v_{2f} - v_{1f} \quad \ldots(3)$$

*The velocity $\vec{v}_{2f}$ has to be greater than velocity $\vec{v}_{1f}$ because otherwise the two colliding bodies cannot separate.

*$(v_{1i} - v_{2i})$ is the magnitude of the relative velocity of A w.r.t. B. **$(v_{2f} - v_{1f})$ is the magnitude of relative velocity of B w.r.t. A. It may be noted that the direction of relative velocity is reversed after the collision.

Relative velocity of A w.r.t. B before collision

= Relative velocity of B w.r.t. A after collision

or Relative velocity of approach = Relative velocity of separation

In one-dimensional elastic collision, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision.

From equation (3), $v_{2f} = v_{1i} - v_{2i} + v_{1f}$

From equation (1), $m_1(v_{1i} - v_{1f}) = m_2(v_{1i} - v_{2i} + v_{1f} - v_{2i})$

or $$m_1v_{1i} - m_1v_{1f} = m_2v_{1i} - 2m_2v_{1f} + m_2v_{1f}$$

or $$-m_1v_{1f} - m_2v_{1f} = -m_1v_{1i} + m_2v_{1i} - 2m_2v_{2i}$$

or $$(m_1 + m_2)\,v_{1f} = (m_1 - m_2)\,v_{1i} + 2m_2v_{2i}$$

or $$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \qquad \text{...(4)}$$

Again, from equation (3), $v_{1f} = v_{2f} - v_{1i} + v_{2i}$

Substituting this value in equation (1) and simplifying, we get

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2}v_{2i} + \frac{2m_1}{m_1 + m_2}v_{1i} \qquad \text{...(5)}$$

Equations (4) and (5) give the final velocities of the colliding bodies in terms of their initial velocities.

3.10. APPLICATION OF ELASTIC COLLISIONS

In a nuclear reactor, the neutrons are produced from the fission of Uranium. These neutrons are very fast. So, they cannot be used to produce more fission. Thus, they have to be quickly slowed down. This is done by making them collide against a target. If the targets are electrons, then the speed of neutrons will remain practically unchanged.

* $(\vec{v}_{1i} - \vec{v}_{2i})$ is the relative velocity of approach.

** $(\vec{v}_{2f} - \vec{v}_{1f})$ is the relative velocity of separation.

This is because neutrons are massive as compared to electrons. If the targets are lead nuclei, then the neutrons merely bounce back with nearly the same speed. This is because neutrons are much lighter than lead nuclei.

If the targets are protons, then the neutrons are sufficiently slowed down because the masses of two colliding particles are comparable.

The protons are available in water. So, water can be used as a moderator in a nuclear reactor. But neutrons tend to constitute stable nuclei with protons. So, instead of water, we use heavy water (D_2O) as moderator. The nucleus of deuterium contains one neutron and one proton only.

Example 3. *Two bodies of masses 50 g and 30 g moving in the same direction, along the same straight line with velocities 50 cm s^{-1} and 30 cm s^{-1} respectively suffer one-dimensional elastic collision. Calculate their velocities after the collision.*

Solution. Mass, m_1 = 50 g ; Mass, m_2 = 30 g ;

Velocity, v_{1i} = 50 cm s^{-1} ; Velocity, v_{2i} = 30 cm s^{-1}

v_{1f} = ?, v_{2f} = ?

$$_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$= \left(\frac{50 - 30}{50 + 30} \times 50 + \frac{2 \times 30}{50 + 30} \times 30 \right) \text{cm s}^{-1} = \mathbf{35\ cm\ s^{-1}}$$

Again,

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i} + \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$= \left(\frac{30 - 50}{50 + 30} \times 30 + \frac{2 \times 50}{50 + 30} \times 50 \right) \text{cm s}^{-1} = \mathbf{55\ cm\ s^{-1}}$$

Example 4. *A body A of mass 2 kg moving with a velocity of 25 m s^{-1} in the east direction collides elastically with another body B of mass 3 kg moving with velocity of 15 m s^{-1} westwards. Calculate the velocity of each ball after the collision.*

Solution. m_1 = 2 kg, v_{1i} = 25 m s^{-1}, m_2 = 3 kg ;

*v_{2i} = – 15 m s^{-1}, v_{1f} = ?, v_{2f} = ?

* The initial velocity of the body B is in a direction opposite to that of the initial velocity of body A.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$= \left(\frac{2-3}{2+3} \times 25 + \frac{2 \times 3}{2+3} \times -15\right) \mathrm{m\,s^{-1}} = \mathbf{-23\ m\ s^{-1}}$$

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i} + \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$= \left(\frac{3-2}{3+2} \times -15 + \frac{2 \times 2}{3+2} \times 25\right) \mathrm{m\,s^{-1}} = \mathbf{17\ m\ s^{-1}}$$

3.11. INELASTIC COLLISION AND COEFFICIENT OF RESTITUTION

The ratio of relative speed of separation after collision and the relative speed of approach before collision is a constant. This constant is called ***coefficient of restitution*** or ***coefficient of resilience.*** It is denoted by *e*. It is a measure of the degree of elasticity of a collision. Its value depends upon the nature of the colliding bodies.

The coefficient of restitution is defined as the ratio of the magnitude of relative velocity of separation after collision to the magnitude of relative velocity of approach before collision.

$$e = \frac{|\vec{v}_{2f} - \vec{v}_{1f}|}{|\vec{v}_{1i} - \vec{v}_{2i}|}$$

(i) In a perfectly elastic collision, the relative velocity of separation is equal to the relative velocity of approach.

$\therefore \qquad e = 1$

Note that there is no loss of kinetic energy. A body dropped from a certain height shall rebound to the same height.

(ii) In a perfectly inelastic collision, the bodies stick together after the collision. The relative velocity of separation is zero.

$\therefore \qquad e = 0$

(iii) In general, the bodies are neither perfectly elastic nor perfectly inelastic. In that case, velocity of separation = e(velocity of approach), where $0 < e < 1$.

For two lead balls, $e = 0.20$ and for the glass balls, $e = 0.95$.

(iv) If $e > 1$, then the collision is superelastic collision. [An example of superelastic collision is that of a cracker which is forcefully struck against the ground.]

3.12. ONE-DIMENSIONAL INELASTIC COLLISION

A collision is said to be one-dimensional inelastic collision if the momentum is conserved with some loss of kinetic energy and the colliding bodies continue to move along the same straight line path before and after the collision.

Consider two bodies A and B of masses m_1 and m_2 moving, in the same direction, along the same straight line path with velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ respectively such that $|\vec{v}_{1i}| > |\vec{v}_{2i}|$. The two bodies A and B undergo head-on collision. After the collision, they continue to move along the same straight line with velocities $\vec{v}_{1f}$ and $\vec{v}_{2f}$ respectively without any change in direction.

Using conservation of momentum,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \ldots(1)$$

If e be the coefficient of restitution, then

$$e = \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$$

or
$$v_{2f} = v_{1f} + e(v_{1i} - v_{2i}) \quad \ldots(2)$$

Substituting the value of v_{2f} in equation (1),

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2[v_{1f} + e(v_{1i} - v_{2i})]$$

or
$$(m_1 + m_2)\, v_{1f} = (m_1 - em_2)\, v_{1i} + (1 + e)\, m_2 v_{2i}$$

or
$$v_{1f} = \frac{m_1 - em_2}{m_1 + m_2} v_{1i} + \frac{(1+e)m_2}{m_1 + m_2} v_{2i} \quad \ldots(3)$$

Similarly,
$$v_{2f} = \frac{m_2 - em_1}{m_1 + m_2} v_{2i} + \frac{(1+e)m_1}{m_1 + m_2} v_{1i} \quad \ldots(4)$$

3.13. LINEAR MOMENTUM OF A SYSTEM OF PARTICLES

Consider a system of n particles of masses m_1, m_2, ..., m_n and velocities $\vec{v}_1$, $\vec{v}_2$, ..., $\vec{v}_n$ respectively. The particles may be interacting and have external forces acting on them. The linear momentum of the first particle is $m_1\vec{v}_1$, of the second particle is $m_2\vec{v}_2$ and so on.

For the system of n particles, the linear momentum of the system is defined to be the vector sum of momenta of all individual particles of the system.

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n$$

But $m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n = M\vec{V}$

$$\therefore \quad \vec{P} = M\vec{V} \qquad \ldots(1)$$

Thus, *the total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.*

3.14. MOMENTUM CONSERVATION

Differentiating Eq. (1) with respect to time,

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{V}}{dt} = M\vec{A}$$

But $\quad M\vec{A} = \vec{F}_{ext.}$

where $\vec{F}_{ext.}$ represents the sum of all external forces acting on the particles of the system.

$$\therefore \quad \frac{d\vec{P}}{dt} = \vec{F}_{ext.} \qquad \ldots(2)$$

This is the statement of **Newton's second law extended to a system of particles.**

Suppose now, that the sum of external forces acting on a system of particles is zero. Then from Eq. (2)

$$\frac{d\vec{P}}{dt} = 0 \quad \text{or} \quad \vec{P} = \text{constant} \qquad ...(3)$$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant. This is the **law of conservation of the total linear momentum** of a system of particles.

Rewriting Eq. (3),

$$M\vec{V} = \text{constant}$$

or $$\vec{V} = \text{constant} \qquad (\because \text{ M is constant.})$$

Thus, if the total external force acting on the system is zero, the centre of mass moves with a constant velocity *i.e.*, moves uniformly in a straight line like a free particle. This is Newton's first law of motion.

3.15. EXAMPLES OF MOTION OF CENTRE OF MASS

Following are examples of motion of centre of mass:

1. A projectile, following the usual parabolic trajectory, explodes into fragments midway in air. The forces leading to the explosion are internal forces. They contribute nothing to the motion of the centre of mass. The total external force, namely, the force of gravity acting on the body, is the same before and after the explosion. The centre of mass under the influence of the external force continues, therefore, along the same parabolic trajectory as it would have followed if there were no explosion.

In this illustration, the forces of explosion are all internal forces. These forces are exerted by part of the system on other parts of the system. These forces may change the momenta of all the individual fragments from the values they had when they made up the projectile. But the internal forces cannot change the total vector momentum of the system. It is

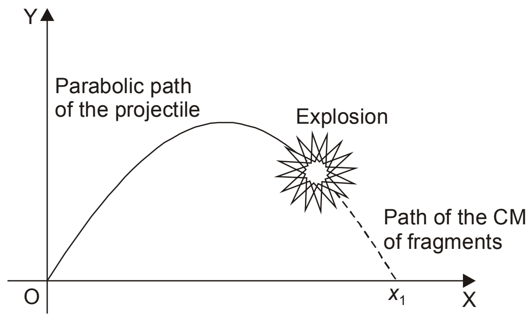


Fig. 3.8. The centre of mass of the fragments of the projectile continues along the same parabolic path which it would have followed if there were no explosion

only the external force which can change the total momentum of the system. In the given problem, the only external force is that due to gravity. The change in the total momentum of the system due to gravity is the same whether the shell explodes or not.

2. Consider the Earth-Moon system. Both the Earth and the Moon move in circles about their centre of mass, always being on opposite sides of it. The centre of mass moves along an elliptical path around the Sun. The forces of attraction between Earth and Moon are internal to the Earth-Moon system. On the other hand, the Sun's attraction of both Earth and Moon are external forces.

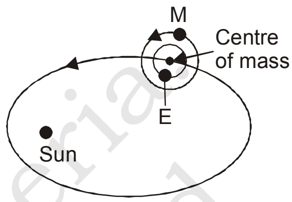


Fig. 3.9. Centre of mass of Earth-Moon system

3.16. RIGID BODIES AND ROTATIONAL MOTION

After having considered a system of particles which moves under the influence of internal and external forces, we can now take up the rotational motion of rigid body. **A rigid body is a body with a perfectly definite and unchanging shape.** *The geometrical shape and size of rigid body do not undergo any change during motion of rigid body.* A rigid body may be regarded as assembly of point masses. The mutual distances among different point masses do not change during the motion of the rigid body.

3.17. CENTRE OF MASS OF A RIGID BODY

*The **centre of mass** of a rigid body is a point whose position is fixed with respect to the body as a whole.* This point may or may not be within the body. The position of the centre of mass of a rigid body depends upon the following two factors.

(i) shape of the body ***(ii)*** distribution of mass in the body.

It is easy to locate the centre of mass of a symmetrical rigid body having uniform distribution of mass. In most of such cases, the centre of mass is at the geometrical centre.

Position of Centre of Mass of Some Regular Bodies

S. No.	Shape of body	Position of centre of mass
1.	Uniform rod	Centre of rod
2.	Plane rectangular or square lamina	Point of intersection of diagonals
3.	Plane triangular lamina	Point of intersection of the medians of triangle
4.	Uniform circular ring	Centre of ring
5.	Uniform circular disc	Centre of disc
6.	Uniform solid sphere	Centre of the solid sphere
7.	Uniform hollow sphere	Centre of the hollow sphere
8.	Uniform hollow cylinder	Midpoint of the axis of the hollow cylinder
9.	Uniform solid cylinder	Midpoint of the axis of the solid cylinder

3.18. LAW OF CONSERVATION OF LINEAR MOMENTUM

(*i*) Statement. ***If the vector sum of the external forces acting on a system is zero, then the total momentum of the system is conserved i.e., remains constant.***

The concept of conservation of momentum is particularly important in situations in which we have two or more interacting bodies. The law of conservation of momentum is a direct consequence of Newton's third law. This law does not depend on the detailed nature of the internal forces that act between the members of the system.

- *For any system of particles, the forces that the particles of the system exert on each other are called internal forces.*
- *The forces exerted on any part of the system by some object outside the system are called external forces.*
- *A system is said to be* ***isolated*** *if the net external force acting on the system is zero.*
- *A system is said to be* ***closed*** *if no particles enter or leave the system.*

For a closed, isolated system,

$$\vec{p} = \text{constant} \quad \ldots(1)$$

$$\Rightarrow \quad \vec{p}_i = \vec{p}_f \quad \ldots(2)$$

The total linear momentum at some initial time t_i = total linear momentum at some later time t_f.

Equations (1) and (2) are vector equations. Each is equivalent to three equations corresponding to the conservation of linear momentum in three mutually perpendicular directions. Depending on the forces acting on a system, linear momentum might be conserved in one or two directions but not in all directions. *If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.*

(*ii*) Derivation of the law of conservation of momentum from Newton's second law of motion.

According to Newton's second law of motion, the time rate of change of momentum is equal to the applied force.

If the system is isolated, then $\vec{F} = 0$.

In that case, $\dfrac{d}{dt}(\vec{p}) = 0$

$\therefore \quad \vec{p} = \text{constant}$

[Differential coefficient of an isolated constant is zero.]

This leads us to the following statement of the law of conservation of momentum.

"In the absence of external forces, the total momentum of the system is conserved".

(*iii*) Derivation of the law of conservation of momentum from Newton's third law of motion.

Consider an isolated system consisting of two bodies A and B of masses m_1 and m_2 respectively [Fig. 3.10]. Let the two bodies be moving along a straight line in the same direction. Let their respective velocities be $\vec{v}_{1i}$ and $\vec{v}_{2i}$ such that $\vec{v}_{1i}$ is greater than $\vec{v}_{2i}$. The two bodies will collide after some time. Let $\vec{v}_{1f}$ and $\vec{v}_{2f}$ be the velocities of A and B respectively after the collision.

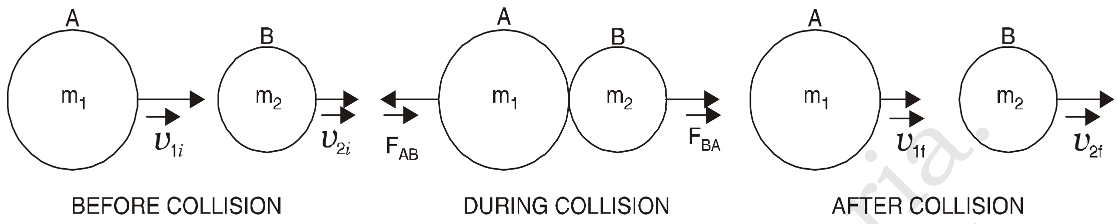


Fig. 3.10. One-dimensional collision

Before collision

Momentum of body A = $m_1\vec{v}_{1f}$; Momentum of body B = $m_2\vec{v}_{2i}$

$\therefore$ Total momentum of system = $m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$

After collision

Momentum of body A = $m_1\vec{v}_{1f}$; Momentum of body B = $m_2\vec{v}_{2f}$

$\therefore$ Total momentum of system = $m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Change in momentum of body A = $m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$

Change in momentum of body B = $m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$

During collision, the body A exerts an average force $\vec{F}_{BA}$ on body B. According to Newton's third law of motion, the body B will exert an average force $\vec{F}_{AB}$ on body A such that

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Let t be the duration of collision.

Then, impulse acting on B = $\vec{F}_{BA}\,t$; Impulse acting on A = $\vec{F}_{AB}\,t$

But impulse = change in momentum

$\therefore$ $\vec{F}_{BA}\,t = m_2\vec{v}_{2i} - m_2\vec{v}_{2f}$ and $\vec{F}_{AB}\,t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$

But $\vec{F}_{BA}\,t = -\vec{F}_{AB}\,t$

$\therefore$ $m_2\vec{v}_{2f} - m_2\vec{v}_{2i} = -(m_1\vec{v}_{1f} - m_1\vec{v}_{1i})$

or $m_2\vec{v}_{2f} + m_1\vec{v}_{1f} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$

So, total momentum of system after collision is equal to the total momentum of system before collision.

This leads to the following statement of the law of conservation of momentum.

"The total vector sum of the momenta of bodies, in an isolated system, along any straight line remains conserved and is unchanged due to the mutual action and reaction between the bodies in the system."

This law is universal. It is true not only for collisions between astronomical bodies but also for collisions between atomic particles.

3.19. APPLICATIONS/ILLUSTRATIONS OF LAW OF CONSERVATION OF MOMENTUM

(*i*) Recoil of a Gun. Let the gun and the bullet in its barrel constitute one isolated system.

To begin with, both the gun and the bullet are at rest. So, the momentum of the system, before firing, is zero.

When the bullet is fired, it moves in the forward direction and the gun kicks backward.

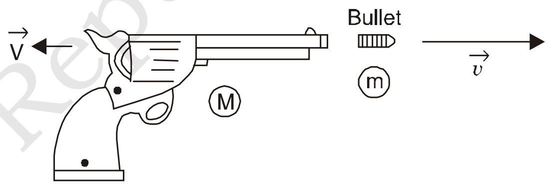


Fig. 3.11. Recoil of gun

Let, m = mass of bullet ; M = mass of gun ; $\vec{v}$ = velocity of bullet ; $\vec{V}$ = velocity of gun.

Total momentum of system after firing = $M\vec{V} + m\vec{v}$

No external forces have acted on the system. So, law of conservation of momentum can be applied.

$$\therefore \qquad M\vec{V} + m\vec{v} = 0 \quad \text{or} \quad M\vec{V} = -m\vec{v}$$

or
$$\vec{V} = -\frac{m}{M}\vec{v}$$

The negative sign shows that the velocity $\vec{V}$ of recoil is opposite to the velocity of the bullet, *i.e.,* if the bullet moves in the forward direction, the gun moves in the backward direction.

The mass M of the gun is very large as compared to the mass m of the bullet. So, the velocity of recoil is very small as compared to the velocity of the bullet.

(*ii*) Machine Gun firing Bullets. Suppose a machine gun mounted on a car is firing n bullets in time t. Let m and $\vec{v}$ be the mass and velocity respectively of each bullet [Fig. 3.12].

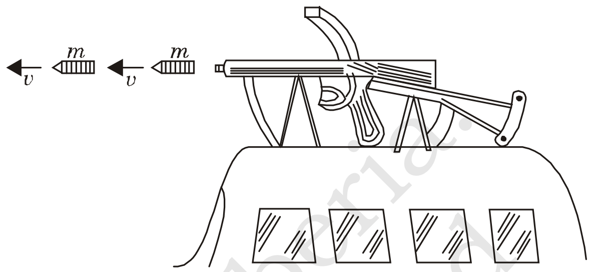


Fig. 3.12. Machine gun firing bullets

Total momentum in the forward direction = $n \times m\vec{v}$

The reaction of this momentum will be in the backward direction. This reaction will set the car in motion to the right. In order to hold the car in position, the accelerator of the car shall have to be suitably pressed. The applied force $\vec{F}$ should be such that

$$\vec{F}\,t = -\,nm\vec{v} \quad \text{[Impulse = change of momentum]}$$

(*iii*) Explosion of a Bomb. Suppose a bomb is at rest as shown in Fig 3.13 (*a*). Its momentum will be zero. Let the bomb explode into five fragments of masses m_1, m_2, m_3, m_4 and m_5 [Fig. 3.13 (*b*)].

Let their respective velocities be $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ and $\vec{v}_5$. Then their respective momenta will be given by

$$\vec{p}_1 = m_1\vec{v}_1,\ \vec{p}_2 = m_2\vec{v}_2,\ \vec{p}_3 = m_3\vec{v}_3,\ \vec{p}_4 = m_4\vec{v}_4 \text{ and } \vec{p}_5 = m_5\vec{v}_5$$

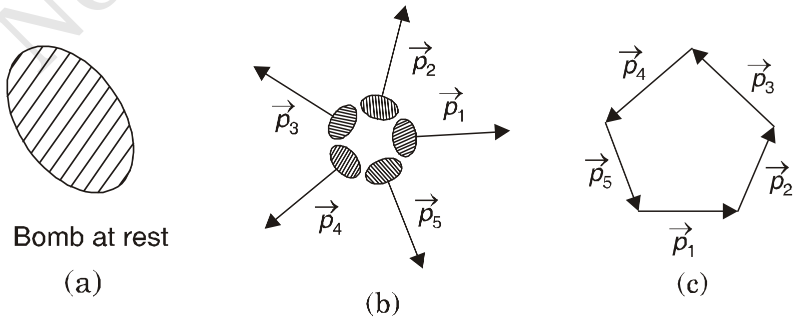


Fig. 3.13. Explosion of a bomb

No external force has acted on the system. Therefore, the law of conservation of momentum can be applied.

$\therefore$ Momentum after explosion = Momentum before explosion

$$\therefore \quad \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \vec{p}_5 = \vec{0}$$

The sum of the five momenta vectors is zero. So, they can be represented both in magnitude and direction by the five sides of a closed polygon, all taken in the same order. This is shown in Fig. 3.13(*c*).

If the bomb explodes into two fragments of equal masses, then the fragments will move with equal speeds in opposite directions.

3.20. LAW OF CONSERVATION OF MOMENTUM AND CENTRE OF MASS

Consider an isolated system consisting of n particles of masses m_1, m_2, m_3, ..., m_n. Let $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, ..., $\vec{v}_n$ be their respective velocities.

The total linear momentum $\vec{P}$ of the system is equal to the vector sum of the linear momenta of all the particles in the system.

Then $\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... + m_n\vec{v}_n$

$$\text{or} \quad \vec{P} = (m_1 + m_2 + m_3 + ... + m_n)\left\{\frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... + m_n\vec{v}_n}{m_1 + m_2 + m_3 + ... + m_n}\right\}$$

But $m_1 + m_2 + m_3 + ... + m_n = \text{M}$ (total mass of system)

$$\text{and} \quad \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... + m_n\vec{v}_n}{m_1 + m_2 + m_3 + ... + m_n} = \vec{V}_{c.m.}$$

where $\vec{V}_{c.m.}$ is the velocity of centre of mass of the system.

$$\therefore \quad \vec{P} = \text{M}\,\vec{V}_{c.m.}$$

Since the given system is isolated therefore no external force will act. According to the law of conservation of momentum, the total momentum $\vec{P}$ should be constant.

$$\therefore \qquad M\vec{V}_{c.m.} = \text{constant.}$$

CONCLUSION

When no external force acts on the system, the centre of mass of the system has a constant velocity.

Example 5. *A gun weighing 10 kg fires a bullet of 30 g with a velocity of 330 m s^{-1}. With what velocity does the gun recoil ? What is the combined momentum of the gun and bullet before firing and after firing?*

Solution. Mass of gun, M = 10 kg

Mass of bullet, m = 30 g = 0.03 kg

Velocity of bullet, v = 330 m s^{-1}

Velocity of recoil, V = ?

In magnitude, momentum of gun = momentum of bullet

$$\therefore \qquad MV = mv \quad \text{or} \quad V = \frac{mv}{M}$$

$$\therefore \qquad V = \frac{0.03 \times 330}{10} \text{ m s}^{-1} = \mathbf{0.99 \text{ m s}^{-1}}$$

Combined momentum of gun and bullet before firing is zero. Since no external force has acted therefore momentum must be conserved. So, the combined momentum of gun and bullet after firing is also **zero.**

Example 6. *A hunter has a machine gun that can fire 50 g bullets with a velocity of 900 m s^{-1}. A 40 kg tiger springs at him with a velocity of 10 m s^{-1}. How many bullets must the hunter fire into the tiger in order to stop him in his track?*

Solution.

Mass of bullet, m = 50 g = 0.05 kg

Velocity of bullet, v = 900 m s^{-1}

Mass of tiger, M = 40 kg

Velocity of tiger, V = 10 m s^{-1}

Let n be the number of bullets required to be pumped into the tiger to stop him in his track.

If the bullets and the tiger are supposed to constitute one isolated system, then the magnitude of the momentum of n bullets should be equal to the magnitude of momentum of the tiger.

$$\therefore \qquad n \times m\,v = \text{MV} \quad \text{or} \quad n = \frac{\text{MV}}{mv}$$

$$\therefore \qquad n = \frac{40 \times 10}{0.05 \times 900} = 8.89 \approx \mathbf{9}$$

3.21. MOMENT OF INERTIA

(*i*) ***Moment of inertia*** *of a rigid body about a fixed axis is defined as the sum of the products of the masses of all the particles constituting the body and the squares of their respective distances from the axis of rotation.* It is a scalar quantity.

Let YY′ be the axis about which the rigid body is rotating [Fig. 3.14]. Let the body be composed of n particles of masses m_1, m_2, ..., m_n. Let r_1, r_2, ..., r_n be their respective distances from the axis of rotation. The moment of inertia of the rigid body about the given axis YY′ is given by

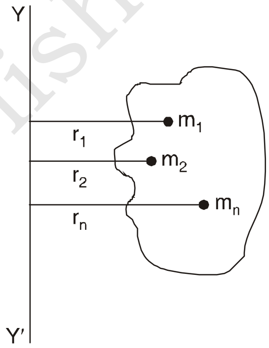


Fig. 3.14. Moment of inertia of a rigid body

$$\text{I} = m_1 r_1^{\,2} + m_2 r_2^{\,2} + \ldots\ldots + m_n r_n^{\,2} = \sum_{i=1}^{n} m_1 r_1^2$$

(*ii*) In cgs system, the unit of moment of inertia is g cm^2. In SI, moment of inertia is measured in kg m^2.

(*iii*) Moment of inertia depends on the following **factors:**

1. Mass of the body.
2. Position of the axis of rotation.
3. Distribution of mass about the axis of rotation.

3.22. ANGULAR MOMENTUM OF A PARTICLE

(*a*) The rotational analogue of momentum is moment of momentum. It is also referred to as angular momentum. This quantity is a measure of the twisting or turning effect associated with the momentum of the particle.

The ***angular momentum (or moment of momentum)*** *about an axis of rotation is a vector quantity, whose magnitude is equal to the product of the magnitude of momentum and the perpendicular distance of the line of action of momentum from the axis of rotation and its direction is perpendicular to the plane containing the momentum and the perpendicular distance.*

Fig. 3.15 shows a particle having linear momentum $\vec{p}$. Its position vector with reference to point O is $\vec{r}$. The perpendicular distance of the line of action of momentum from O is d. The angular momentum of the particle about an axis passing through O and perpendicular to the plane of the paper is given by:

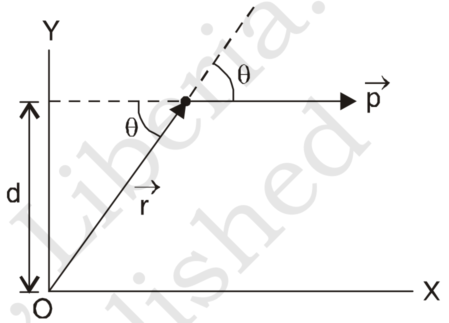


Fig. 3.15

$$L = pd$$

The cgs and SI units of L are g cm^2 s^{-1} and kg m^2 s^{-1} respectively. Its dimensional formula is [ML^2T^{-1}].

(*b*) Angular Momentum in Vector Notation. Fig. 3.16 shows position vector $\vec{r}$ and momentum $\vec{p}$ of a particle P in XOY plane. The angular momentum of the particle P with respect to the origin O is given by:

$$\vec{L} = \vec{r} \times \vec{p}$$

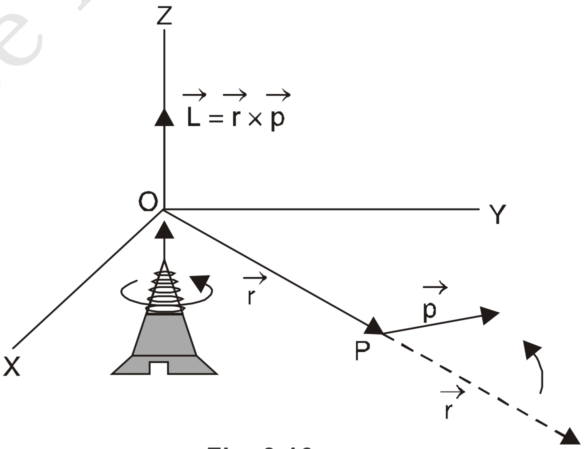


Fig. 3.16

The direction of $\vec{L}$ is obtained by applying the right-hand rule for the vector product of two vectors. In this case, $\vec{L}$ acts along OZ.

The angular momentum is taken as positive for anti-clockwise rotation and negative for clockwise rotation.

The magnitude of $\vec{L}$ is given by, $L = rp \sin\theta$ ...(1)

where r is the magnitude of the position vector $\vec{r}$ *i.e.*, the length OP, p is the magnitude of momentum $\vec{p}$ and θ is the angle between $\vec{r}$ and $\vec{p}$ as shown.

Now, $\sin\theta = \dfrac{d}{r}$ or $d = r\sin\theta$

From eqn. (1), $L = p\,(r\sin\theta) = pr_{\perp} = pd$

Again, $L = r\,(p\sin\theta) = rp_{\perp} = rp_{\theta}$

Special Cases

(*i*) If $r = 0$, then L = 0. A particle at O has zero angular momentum about O.

(*ii*) If $\theta = 0°$ or $180°$, then $\sin\theta = 0$.

$\therefore$ $L = rp\sin\theta = 0$

In this case, the line of action of the momentum passes through the point O. Thus, if the line of action of momentum passes through point O, the angular momentum is zero.

(*iii*) If $\theta = 90°$, then $\sin\theta = \sin 90° = 1$ (max. value). So, L is maximum.

$L_{max.} = rp$

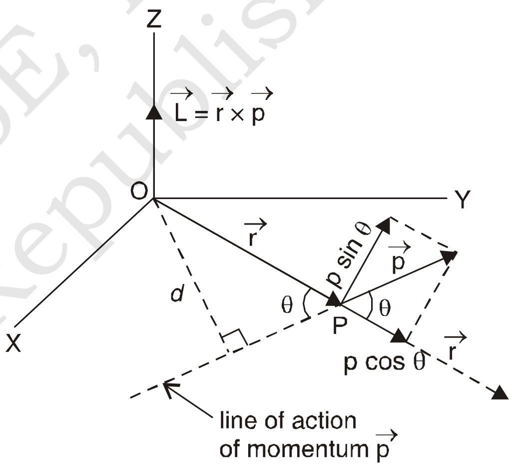


Fig. 3.17

3.23. RELATION BETWEEN ANGULAR MOMENTUM AND TORQUE

We know that, $\vec{L} = \vec{r} \times \vec{p}$

Differentiating both sides w.r.t. t, we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

or

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \vec{v} \times \vec{p}$$

or $$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} \quad [\because \vec{v} \times \vec{p} = \vec{v} \times m\vec{v} = m(\vec{v} \times \vec{v}) = 0]$$

According to Newton's second law of motion, $\frac{d\vec{p}}{dt} = \vec{F}$

$$\therefore \quad \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \quad \text{or} \quad \frac{d\vec{L}}{dt} = \vec{\tau} \qquad \text{...(1)}$$

So, *the time rate of change of the angular momentum of a particle is equal to the torque acting on it.* This result is the rotational analogue of the statement—"The time rate of change of the linear momentum of a particle is equal to the force acting on it."

Like all vector equations, equation (1) is equivalent to three scalar equations, namely, $\tau_x = \frac{dL_x}{dt}$, $\tau_y = \frac{dL_y}{dt}$ and $\tau_z = \frac{dL_z}{dt}$

So, the x-component of the applied torque is given by x-component of the change with time of the angular momentum. Similar results hold for the y and z-directions.

3.24. TORQUE AND ANGULAR MOMENTUM FOR A SYSTEM OF PARTICLES

The total angular momentum of a system of particles about a given point is the vector sum of the angular momenta of individual particles about the given point. For a system of n particles,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \ldots\ldots + \vec{L}_n = \sum_{i=1}^{n} \vec{L}_i$$

The angular momentum of the i^{th} particle is given by

$$\vec{L}_i = \vec{r}_i \times \vec{p}_i$$

where $\vec{r}_i$ is the position vector of the i^{th} particle with respect to the given origin and $\vec{p}_i$ $(= m_i\vec{v}_i)$ is the linear momentum of the i^{th} particle.

Now, $$\vec{L} = \sum_{i=1}^{n} \vec{L}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i$$

This is a generalisation of the definition of angular momentum for a single particle to a system of particles.

Now,
$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\Sigma\vec{L}_i) = \sum_{i=1}^{n}\frac{d\vec{L}_i}{dt} = \sum_{i=1}^{n}\vec{\tau}_i$$

where $\vec{\tau}_i$ is the torque acting on the i^{th} particle;

3.25. EQUILIBRIUM OF RIGID BODIES

A rigid body is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time, or equivalently the body has neither linear acceleration nor angular acceleration.

A rigid body such as a chair, a bridge or building is said to be in equilibrium if both the linear momentum and the angular momentum of the rigid body have a constant value. When a rigid body is in equilibrium, the linear acceleration of its centre of mass is zero. Also, the angular acceleration of the rigid body about any fixed axis in the reference frame is zero.

For the equilibrium of a rigid body, it is not necessary that the rigid body is at rest. However, *if the rigid body is at rest, then the equilibrium of the rigid body is called* ***static equilibrium.***

(*i*) First Condition for Equilibrium. The translational motion of the centre of mass of a rigid body is governed by the following equation :

$$\Sigma\vec{F}_{ext.} = \frac{d}{dt}(\vec{p})$$

A rigid body is said to be in translational equilibrium if it remains at rest or moves with a constant velocity in a particular direction.

Here $\Sigma\vec{F}_{ext.}$ is the vector sum of all the external forces that act on the rigid body.

For equilibrium, $\vec{p}$ must have a constant value. $\therefore \frac{d}{dt}(\vec{p}) = 0$

$$\therefore \qquad \Sigma\vec{F}_{ext.} = 0$$

This vector equation is equivalent to three scalar equations:

$$\sum_{i=1}^{n} F_{ix} = 0, \sum_{i=1}^{n} F_{iy} = 0, \sum_{i=1}^{n} F_{iz} = 0 \qquad ...(1)$$

This leads us to ***the first condition for the equilibrium of rigid bodies.***

"The vector sum of all the external forces acting on the rigid body must be zero".

(*ii*) Second Condition for Equilibrium. The rotational motion of a rigid body is governed by the following equation:

$$\Sigma \vec{\tau}_{ext.} = \frac{d\vec{L}}{dt}$$

Here $\Sigma \vec{\tau}_{ext.}$ represents the vector sum of all the external torques that act on the body.

For equilibrium, $\vec{L}$ must have a constant value. $\therefore \frac{d}{dt}(\vec{L}) = 0$

$$\therefore \qquad \Sigma \vec{\tau}_{ext.} = 0$$

This vector equation can be written as three scalar equations:

$$\sum_{i=1}^{n} \tau_{ix} = 0, \sum_{i=1}^{n} \tau_{iy} = 0, \sum_{i=1}^{n} \tau_{iz} = 0 \qquad ...(2)$$

This leads us to ***the second condition for the equilibrium of rigid bodies.***

"The vector sum of all the external torques acting on the rigid body must be zero."

3.26. SOME APPLICATIONS AND EXAMPLES OF THE LAW OF CONSERVATION OF ANGULAR MOMENTUM

1. The angular velocity of a planet around the Sun increases when it comes near the Sun.

When a planet revolving around the Sun in an elliptical orbit comes near the Sun, the moment of inertia of the planet about the Sun decreases. In order to conserve angular momentum, the angular velocity shall increase. Similarly, when the planet is away from the Sun, there will be a decrease in the angular velocity.

2. The speed of the inner layers of the whirlwind in a tornado is alarmingly high.

In a tornado, the moment of inertia of air will go on decreasing as the air moves towards the centre. This will be accompanied by an increase in angular velocity such that the angular momentum is conserved.

3. A diver jumping from a spring board performs somersaults in air.

When a diver jumps from spring board, he curls his body by rolling in his arms and legs. This decreases moment of inertia and hence increases angular velocity. He then performs somersaults. As the diver is about to touch the surface of water, he stretches out his limbs. By so doing, he increases his moment of inertia, thereby reducing his angular velocity.

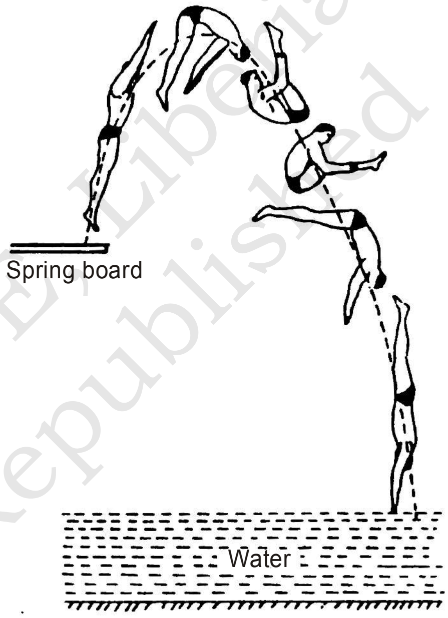


Fig. 3.18. Diver performing somersaults

4. A ballet dancer can vary her angular speed by outstretching her arms and legs.

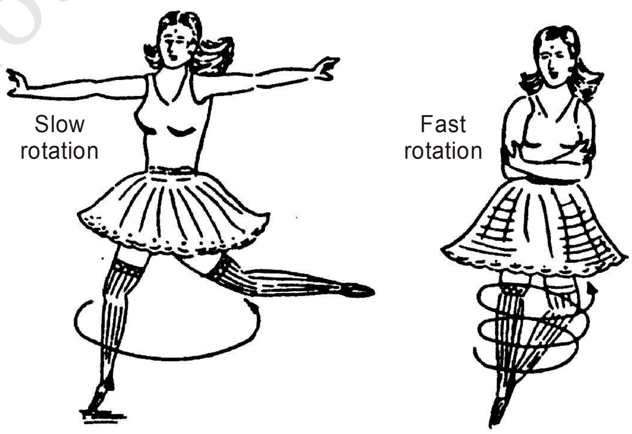


Fig. 3.19. Ballet dancer making use of law of conservation of angular momentum

A ballet dancer [Fig. 3.19] makes use of the law of conservation of angular momentum to vary her angular speed. Suppose a ballet dancer is rotating with her legs and arms stretched outwards. When she suddenly folds her arms and brings the stretched leg close to the other leg, her angular velocity increases on account of decrease in moment of inertia [Fig. 3.19].

5. A man carrying heavy weights in his hands and standing on a rotating table can vary the speed of the table.

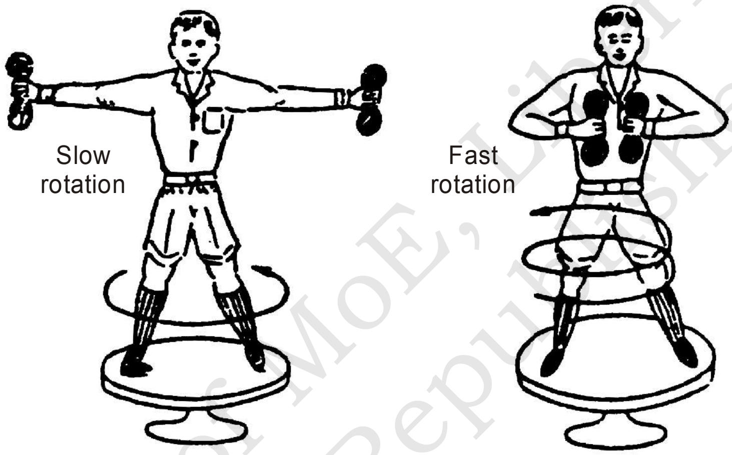


Fig. 3.20

Suppose a man is standing on a rotating table with his arms outstretched. Suppose he is holding heavy weights in his hands. When the man suddenly folds his arms, his angular velocity increases on account of the decrease in moment of inertia [Fig. 3.20].

3.27. LAWS OF ROTATIONAL MOTION

I. Unless an external torque is applied to it, a body in a state of rest or uniform rotational motion about its fixed axis of rotation remains unchanged.

II. The rate of change of angular momentum of a body about a fixed axis of rotation is directly proportional to the torque applied and takes place in the direction of the torque.

III. When a torque is applied by one body on another, an equal and opposite torque is applied by the latter on the former about the same axis of rotation.

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A ball of mass M falls from a height h on a floor for which the coefficient of restitution is e. The height attained by the ball after two rebounds is

(*a*) $e^2 h$ (*b*) eh^2

(*c*) $e^4 h$ (*d*) h/e^4.

2. Consider the following two statements:

A. Linear momentum of a system of particle is zero. Then

B. Kinetic energy of a system of particles is zero. Then

(*a*) A does not imply B but B implies A.

(*b*) A implies B and B implies A.

(*c*) A does not imply B and B does not imply A.

(*d*) A implies B but B does not imply A.

3. A spring of spring constant 5×10^3 N m^{-1} is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(*a*) 25.00 N m (*b*) 6.25 N m

(*c*) 12.50 N m (*d*) 18.75 N m.

4. A neutron makes a head-on elastic collision with a stationary deuteron. The fractional energy loss of the neutron in the collision is

(*a*) 16/81 (*b*) 8/9

(*c*) 8/27 (*d*) 2/3.

5. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is

(*a*) m_2/m_1 (*b*) m_1/m_2

(*c*) 1 (*d*) m_1v_2/m_2v_1.

6. A body of mass m has a kinetic energy equal to one-fourth kinetic energy of another body of mass $m/4$. If the speed of the heavier body is increased by 4 m s^{-1}, its new kinetic energy equals the original kinetic energy of the lighter body. The original speed of the heavier body in m s^{-1} is

(*a*) 8 (*b*) 6

(*c*) 4 (*d*) 2.

7. A toy gun has a spring of force constant *k*. After charging the spring by compressing it through a distance of *x*, the toy releases a shot of mass *m* vertically upwards. Then the shot will travel a vertical height of

(*a*) $\frac{2mg}{kx^2}$ (*b*) $\frac{kx^2}{mg}$

(*c*) $\frac{kx}{mg}$ (*d*) $\frac{kx^2}{2mg}$.

8. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement *x* is proportional to

(*a*) x^2 (*b*) e^x

(*c*) x (*d*) $\log_e x$.

9. An automobile travelling with a speed of 60 km h^{-1}, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e.*, at 120 km h^{-1}, the stopping distance will be

(*a*) 20 m (*b*) 40 m

(*c*) 60 m (*d*) 80 m.

10. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ?

(*a*) 7.2 J (*b*) 3.6 J

(*c*) 120 J (*d*) 1200 J.

B. Fill in the Blanks

1. A body of mass 3 kg is under a constant force which causes a displacement *s* (in m) in it, given by the relation $s = \frac{1}{3}t^2$, where *t* is in second. Work done by the force in 2 *s* is __________ .

2. A 2 kg block slides on a horizontal floor with a speed of 4 m s^{-1}. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N m^{-1}. The spring compresses by __________ .

3. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is __________ .
4. A bread gives a boy of mass 40 kg an energy of 21 kJ. If the efficiency is 28%, then the height which can be climbed by him using this energy is nearly __________ .
5. A windmill converts wind energy into electrical energy. If v is the wind speed, electrical power output is proportional to __________ .

C. Very Short Answer Questions

1. A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy?
2. Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?
3. A body is being raised to a height h from the surface of earth. What is the sign of work done by
   (*a*) applied force
   (*b*) gravitational force?
4. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m.
5. A body falls towards Earth in air. Will its total mechanical energy be conserved during the fall? Justify.

D. Short Answer Questions

1. The potential energy of two atoms separated by a distance x is given by $U = -\frac{A}{x^6}$ where A is a positive constant. Find the force exerted by one atom on another atom.
2. A ball, dropped from a height of 8 m, hits the ground and bounces back to a height of 6 m only. Calculate the fractional loss in kinetic energy.
3. A particle of moving in a circle with centripetal force $-\frac{K}{r^2}$. What is the total energy associated?

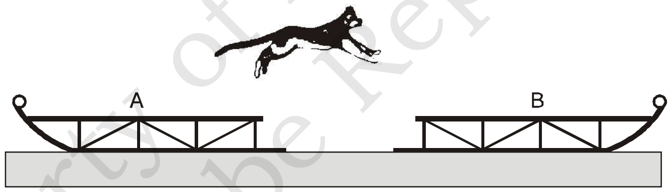
4. A particle of mass m strikes on ground with angle of incidence 45°. If coefficient of restitution $e = 1/\sqrt{2}$, find the velocity of reflection and angle of reflection?

5. A body of mass m falls from a height h and collides with another body of same mass. After collision, the two bodies combine and move through distance d till they come to rest. Find the work done against the resistive force.

E. Long Answer Questions

1. A rubber ball of mass 50 g falls from a height of 1 m and rebounds to a height of 50 cm. Calculate the impulse and the average force between the ball and the ground, if the time during which they are in contact was 0.1 second.

2. Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in figure below. A 3.63 kg cat, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 m s^{-1} relative to the ice. Find the final speeds of the two sleds.



3. A bullet of mass 7 g is fired into a block of metal weighing 7 kg. The block is free to move. After the impact, the velocity of the bullet and the block is 0.7 m s^{-1}. What is the initial velocity of the bullet ?

4. A block of mass m moving at speed v collides with another block of mass 2 m at rest. The lighter block comes to rest after the collision. Find the coefficient of restitution.

5. Two bodies of masses m_1 and m_2 ($< m_1$) are connected to the ends of a massless cord and allowed to move as shown. The pulley is both massless and frictionless. Determine the acceleration of the centre of mass.